More Drag Fun

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Let's take a look at that third postlab question. We'll simulate the drop using Python and compare the values of n when we start "clocking" the coffee filter from different heights. First, we'll derive some of the mathematics useful for our investigations, and then we'll do the actual simulation.

1 The Math

Without proof, we'll state here that the velocity of an object falling with drag force $F_D = -kv^2$ is:

$$v = \sqrt{\frac{mg}{k}} \tanh \sqrt{\frac{kg}{m}} t \tag{1}$$

which, just by integration, implies that its position is:

$$x = \frac{m}{k} \ln \left(\cosh \sqrt{\frac{kg}{m}} t \right) \tag{2}$$

We can easily invert this to find time as a function of position:

$$t = \sqrt{\frac{m}{kg}} \cosh^{-1} \left(e^{kx/m} \right) \tag{3}$$

This should be enough math to get us through our simulation.

2 The Simulation

What we want to do is pretend that the coffee filter is a falling particle, subject only to a quadratic drag force F_D and gravity mg. So just what are we simulating? In the lab, what we ultimately recorded as our data were the mass of the coffee filter stack and the time taken to fall the last 2.26 m to the ground. So, we'll use the recorded masses as our independent variable, and we'll compute this elapsed time (or at least what *should* be the elapsed time) using our equations. With equation 3, we can compute the elapsed time as the difference between the time at position $x = h_0$ (the height at which we start recording) and $x = h_0 + 2.26$ m (the height at which we stop recording, or the floor). Pretending that the ladder is exactly 12 feet tall (3.66 m), this means that $h_0 = 3.66$ m $- 2.26$ m $= 1.40$ m. We can then compute our experimental velocity as $\bar{v} = \Delta h/\Delta t = 2.26/\Delta t$.

As we did in the lab, we can linearize the equation by taking the log of both mg and $\bar{v}$. Taking the mass values measured in the lab, this gives us the graph, with regression line included:

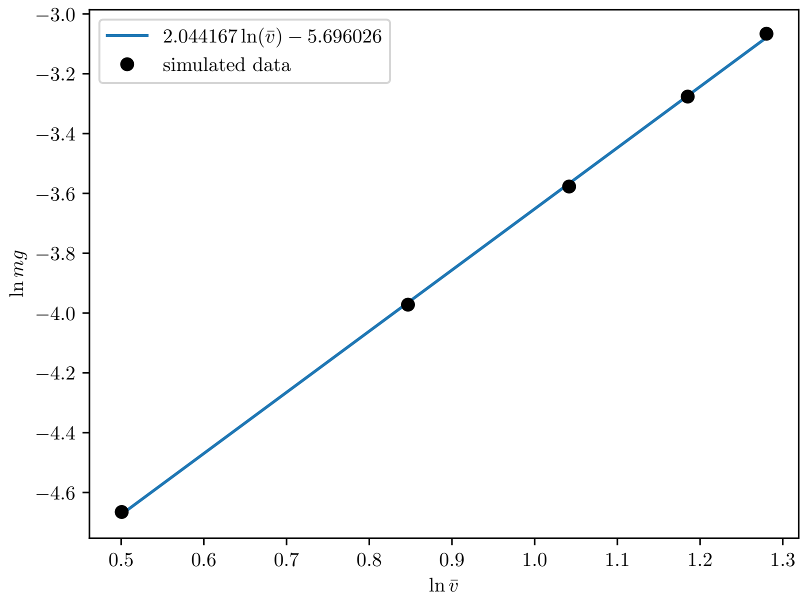

Figure 1: Simulation of dropping a coffee filter and starting the clock from a height of 2.26 m

Let's back up and consider the original analysis question: if we started the clock too soon (i.e. at some height > 2.26 m), what would happen to the value of n in our regression? Well, to make this clear, let's simulate what would happen if we were to start the clock all the way at the top (i.e. height 3.66 m). Using the same mass values and doing the same regression, we get:

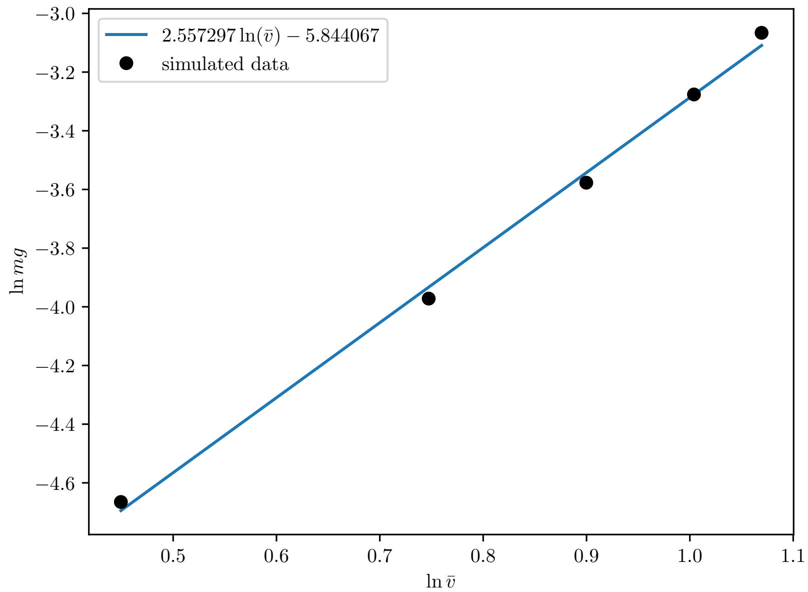

Figure 2: Simulation of dropping a coffee filter and starting the clock from a height of 3.66 m

It looks like n has increased from 2.04 to 2.56! I really need to go do my homework now, so I'll stop this writeup here. If you want to take a look at the source code for the simulation, just ask me or something. Maybe I'll even put it up on my website once I scrub it clean.